Lecture 12

Vocabulary – read the textbook Glossary to learn more about these words!

- fixed annuity
- Individual Retirement Account
- Regulation Q
- Savings Bond
- Section 529 plan
- Single-Premium Deferred Annuity
- tax-deferred
- tax equivalent yield
- tax-exempt bond
- term life insurance
- Treasury Inflation-Protected Security
- variable universal life
- whole life

Daily problems – after this lecture the problems you should master include:

- Exercises 5.3A, page 162, #1 - 7
- Exercises 5.3B, page 164, #1 - 3
- Exercises 5.4A, page 166, #1 – 5
- Exercises 5.4B, p. 171, #1 – 3. 5, 6

Street-Bite

Savings vehicles

- Most important fact: a disciplined savings plan, preferably an automatic plan, is an effective way to accumulate wealth
- Roth IRA uses after-tax money for investing. Wealth accumulates without periodic taxes. Upon withdrawing the accumulation (5-years minimum or else penalties apply) taxes are paid on the gains only. Annual limit on the contribution is $3,000 per household.
- Traditional IRA uses pretax money for investing. Wealth accumulates without periodic taxes. Upon withdrawing the accumulation (59.5 years of age-minimum or else penalties apply) taxes are paid on the entire withdrawal. If covered by an employer 401k plan then you are ineligible for a traditional IRA.
- When the rates of return and lifetime tax rates are the same for the traditional and Roth IRA then they both accumulate the same amount of after-tax wealth. The differences depend on flexibility for using the money and for starting a new plan the Roth is generally more flexible.

Discussion

Chapter 5: Future and Present Values of Annuities

Section 3. Present values of annuities

- Formula 5.1 may be the most important in finance
  - definition 5.2 for *PVIFA*
  - understand the signage

3.A. Beginning wealth, *PV*, as the unknown variable

- simplest case is with $PV = 0$

PROBLEM: Find *PV* and total interest [for annuities we will not partition into interest-on-interest and interest-on-principal] (Exercises 5.3A, page 162, #2 PV5)

- more complex case is with $PV \neq 0$

PROBLEM: Suppose in the preceding problem there was a one-time back-end cash flow of $10,000. Find *PV*.

- Concatenate annuity streams and use the *CF* menu on the calculator

PROBLEM: Find actual *ROR* given two-step annuity stream, target ROR, and actual *PV* PV3

PROBLEM: Find *ROR* given annuity with beginning and ending balance (Exercises 5.3A, page 162, #7 PV11a,b)

3.B. The special case of perpetuities
- Definition 5.3 for perpetuity and formula 5.3

PROBLEM: Find *PV* for an endowment (Exercises 5.3B, page 164, #2 PV12)

Section 4. Cash flows connecting beginning and ending wealths
- Every formula in this chapter, such as formulas 5.4 and 5.5, are simply rearrangements of formula 5.1

EXAMPLE 12, page 168: Saving young versus saving later

FORMULA 5.1 Constant annuity time value formula

$$PV = \frac{CF}{(1+r)^1} + \frac{CF}{(1+r)^2} + \ldots + \frac{CF}{(1+r)^N} + \frac{FV}{(1+r)^N}$$

.

$$= (CF)\left\{\frac{1-(1+r)^{-N}}{r}\right\} + FV(1+r)^{-N}$$

DEFINITION 5.2 Present value interest factor of annuities (PVIFA)

PVIFA is the initial deposit earning interest at the periodic rate *r* that perfectly finances a series of *N* consecutive one dollar withdrawals:

$$PVIFA_{r,N} = \frac{1-(1+r)^{-N}}{r} \quad .$$

EXERCISES 5.3A

2. You're quite fortunate because this afternoon, just like this date in each of the past 10 years, you shall withdraw $1,600 from an account that your guardian angel established for you exactly 11 years ago. After the withdrawal the balance will equal zero. The account earns 6.30% per year (compounded annually, interest is being credited this morning). Except for your 11 withdrawals the account has been untouched. Find the initial deposit that your guardian angel used to establish the account. ©PV5

The following set-up pertains to the next few questions
Multiple setup (PV3m)
You might invest in an asset that will return after-tax cash flow to you of $2,200 per month for 5 months (first payment one month from now), followed by $3,500 per month for 4 months . You make an offer to buy the asset so that you'll get your "target" annual rate of return of 15.7% (compounded monthly).
{CLUES: pv(inflows)=$23,282 }

PV3bm Find PV given an investment's return stream and target rate of return
What is your offer price?
{ANSWER: $23,282

PV3cm Find the actual ROR given an investment's return stream, target ROR, and counteroffer purchase price

Instead, however, the seller makes a counter-offer that is $950 higher than your offer. If you buy at the counter-offer price, and receive the expected cash flows, what is your annual rate of return?

{ANSWER: 6.8%}

EXERCISES 5.3A

7. You might purchase an investment that incurs a large up-front cost today. Furthermore, it requires payments of $2,400 per month for 15 months (first payment one month from now). Immediately after making the last payment, however, you will receive after-tax net proceeds of $78,900. You make an offer to purchase the asset so that you'll get your "target" annual rate of return of 20.30% (compounded monthly).

7a. Find the up-front purchase price that you offer to pay today. ©PV11am .

7b. The seller makes a counteroffer that is $6,000 higher than your offered purchase price. Find your annual rate of return if you buy at the counteroffer price and receive the expected cash flows. ©PV11bm .

DEFINITION 5.3 PERPETUITY

A *perpetuity* is an account that maintains a specified principal balance perpetually even in the absence of subsequent deposits.

The balance never diminishes because all withdrawals consist exclusively of interest, not principal. For example, in the preceding illustration the balance begins at $1 million. Exactly one year later, immediately before the first withdrawal, the account earns periodic interest of $100,000 and the balance rises to $1,100,000. Then the $100,000 withdrawal occurs, the balance falls back to $1 million. The cycle repeats perpetually.

The perpetuity is a special case of the constant annuity time value relation shown in formula 5.1. Mathematically speaking, with a perpetuity N goes to infinity. As N gets larger and larger, the expression $(1+r)^N$ gets larger and larger, too (as long as $r > 0$, actually). Dividing this ever larger number into a future sum causes the present value of that sum to vanish. Cash flows way out yonder have virtually zero effect on present value – it's convergent!

The perpetuity formula relates beginning wealth, periodic cash flow, and rate of return as follows:

FORMULA 5.3 Present value of a perpetual and constant stream

$$PV = \frac{CF}{r} \quad .$$

Variable definitions are the same as always. *PV* is the beginning balance, *CF* is the periodic cash flow. The periodic interest rate *r* equals the annual percentage rate *i* divided by *m,* the number of compounding periods per year.

EXERCISES 5.3B

2. An alumni group wants to establish an endowment fund for paying expenses associated with hiring a distinguished professor of business. The annual expenses should run about $140,000 (payable in 12 monthly installments). Find the size of the requisite endowment if the account earns 8.8% compounded monthly. ©PV12 .

EXAMPLE 12 Saving young versus saving later

Your two twin sisters, Prudence and Candy, are pursuing two different financial strategies for early retirement. Both sisters intend to retire exactly 30 years from today. Candy does not want to start saving until exactly 10 years from today, at which time she'll make her first monthly deposit of $250. She'll continue making monthly deposits for 20 years, so that Candy's final deposit occurs exactly 30 years from today when she retires. The other sister, Prudence, plans to deposit $250 per month for 10 years with the first deposit today and the last one exactly 10 years from today. Prudence will not deposit anything beyond that, but she will let interest continue to accrue. The annual savings rate always is 12% compounded monthly. Both sisters intend to draw down the savings accounts to zero by making monthly withdrawals during retirement for 25 years. The first withdrawal is one month after retirement commences. How much monthly income should each sister expect in retirement?

FOR CANDY:

		10 years		30 years		55 years
t = 0	1	120	121	360	361	660 months
		Dep #1	Dep #2	Dep #241	Wdw #1	Wdw #300
CF= $0	*CF*= $0	*CF*= $-250	*CF*= $-250	*CF*= $-250		
				FV = $x		
				PV = $x	*CF = ?*	*CF = ?*

This future value shown on the time line as *$x* is $250,037. Candy makes wise investments, saving $250 per month for twenty years enabling her to retire on monthly income of $2,633 for twenty-five years. The power of compound interest is amazing!

FOR PRUDENCE. She saves immediately, as shown in the time line below.

		10 years		30 years		55 years
t = 0	1	120	121	360	361	660 months
Dep #1	Dep #2	Dep #121			Wdw #1	Wdw #300
CF= $-250	*CF*= $-250	*CF*= $-250				
		FV = $y				
		PV = $y		*FV = $z*		
				PV = $z	*CF = ?*	*CF = ?*

This future value shown on the time line as *$y* is $58,335. The time line shows *$Z* which computes as $635,415. Prudence saves $250 per month for ten years enabling her to retire on monthly income of $6,692 for twenty-five years.

The difference between Prudence and Candy reflects the TIME VALUE OF MONEY! Prudence saves half as much principal as her sister Candy. Nonetheless, Prudence retires on two-and-a-half times as much income. Total accumulated interest, and not principal, represents almost all the money on which these retirees will live. The money made by their labor is less than the money made by their money!